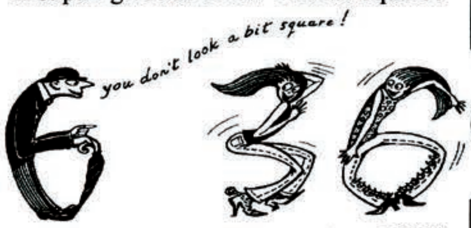
The equations in ***m*** and ***n*** give

$$m = (21 + 1)/2 = 11 \text{ and } n = (21 - 1)/2 = 10$$

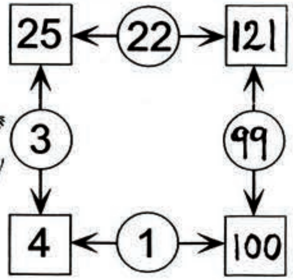
or

$$m = (7 + 3)/2 = 5 \quad \text{and } n = (7 - 3)/2 = 2$$

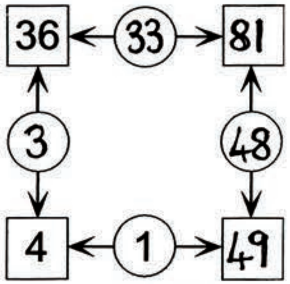
. . but the latter pair repeat the same square numbers 25 and 4 that we began with, so only the first pair gives us a new 'Perfect Square':



(Shutl, PiE no. 92, Spring 1981)



Any 'Perfect Square' with these four corners would have to have a number smaller than 4 on the bottom edge: but 2 would give us a repeat of 2 on the left edge and so would be discounted, leaving 3 as the only other option. The final 'Perfect Square' is



a) how many other 'Perfect Squares' have corners 4,36,81,49?

b) Prove that in a 'Perfect Square' with corners a^2, b^2, m^2, n^2,

$$m^2 - n^2 = b^2 - a^2$$

c) How many 'Perfect Squares' are there with corners < 20^2 ?

How Many Dots?

Another open-ended/discussion piece. 24, of course, but it's fun looking for different possible easily recognisable groupings as well as the obvious approach of counting rows or columns.

Cashback

Jack needs a minimum of eight coins;

50p, 20p, 20p, 10p, 5p, 2p, 2p, 1p or

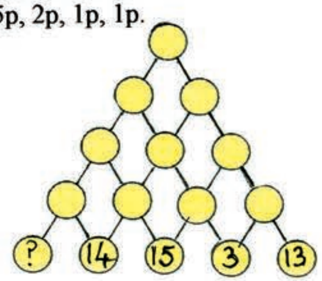
50p, 20p, 10p, 10p, 5p, 2p, 2p, 1p or

50p, 20p, 20p, 10p, 5p, 2p, 1p, 1p or

50p, 20p, 10p, 10p, 5p, 2p, 1p, 1p.

P.S. - Triangle Puzzle:

Here is the basis for one solution to a fifteen - number triangle for you to complete.
Send in any others you find!
Is it possible to 'build up' triangles using sums instead of differences?



MAthematical PiE

Autumn 2019 Notes for . . . . . . No. 208

Total Knowledge

I think that with puzzles of this kind there is great pleasure in discovering strategies for oneself, so I am reluctant to 'give the game away': however for those who would feel cheated without them, here are some hints and the solutions. Consider the possible 'contributions' to the totals that particular polyominoes can make. For example, if a pentomino has three cells in a particular row then the maximum contibution it could make would be 5 + 4 + 3 = 12, or the minimum would be 1 + 2 + 3 = 6. Look at the largest and smallest totals.

This is epitomised well in the second column of the first puzzle: the minimum that the square tetromino can contibute to the total of 6 is 1 + 2 = 3. Each of the three other cells in this column must therefore contain 1. By a converse reasoning, the top and bottom cells of the third column must contain 4 (and the cell of the pentomino that is in the fourth column must now contain 2 since 1, 3, 4 and 5 are already accounted for). Of course as yet we do not know the precise positions of the 1 and 2 in the square or the 3, 4 and 5 in the pentomuino but these will become clear after similar reasoning with some of the rows.

The middle and later stages involve a certain degree of 'suck it and see'; 'what would happen if (say) 2 were in this cell?' (considering the implications for cells in the same row or column).

3	1	4	2	3	13
4	1	5	2	4	16
2	1	3	1	3	10
3	1	4	2	2	12
4	2	4	1	3	14
16	6	20	8	15	

1	5	2	3	1	12
3	4	4	4	3	18
2	4	3	4	2	15
1	3	1	2	1	8
2	4	2	3	1	12
9	20	12	16	8	

4	3	4	2	3	16
2	1	3	1	3	10
1	1	2	1	1	6
4	4	5	4	4	21
3	2	3	2	2	12
14	11	17	10	13	

3	2	4	2	2	13
1	1	2	1	1	6
4	3	4	3	3	17
4	3	5	4	4	20
2	1	3	1	2	9
14	10	18	11	12	

Two-word Phrases

BAR CHART, SQUARE ROOT, RIGHT ANGLE, PRIME NUMBER. The remaining letters are an anagram of STRAIGHT LINE. (I confess I used a crossword-solver for this answer!)

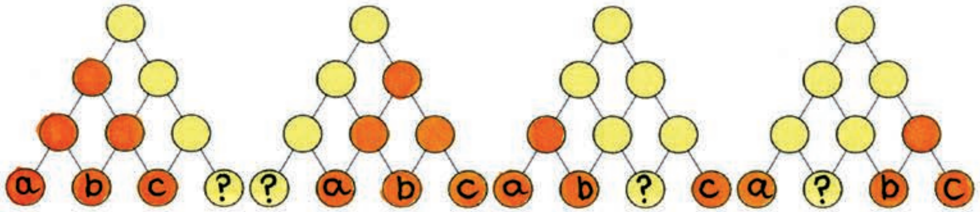
It's a Giveaway

One-quarter of the money is left so she started with less than £120000 and has given less than £90000. The total recieved by each grandchild in the ten years is

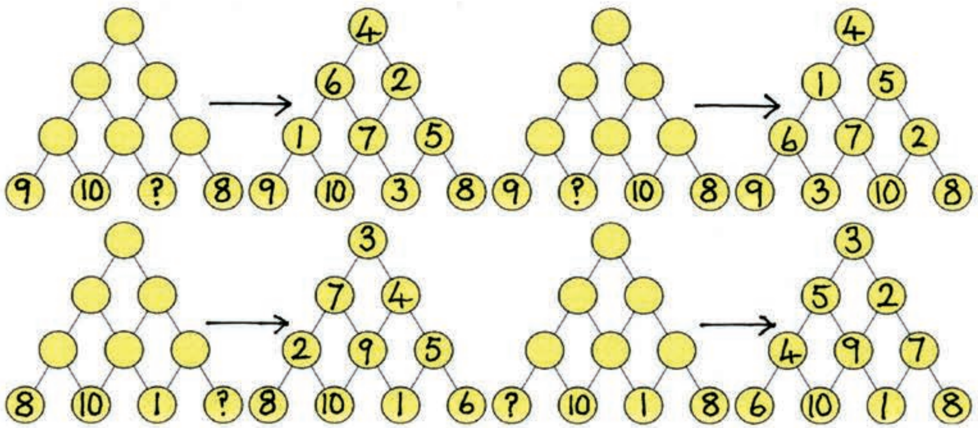
(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10) x £500 = £17,500. Since this divides into £90000 just over 5 times, there must be 5 grandchildren and she has given £87500 in total.

Triangle Puzzle

Finding a strategy to get all the solutions is not easy. 10 has to be on the bottom row but 9 and 8 could be on the bottom row or possibly the row above it. So we need to try all the possible rearrangements of {8, 9, 10}, {(10 with 1) and 8}, {(9 with 1) and 10}or {(10 with 2) and 9} in place of a,b,c in these formats:



After completing as many differences as possible, place one of the remaining numbers in the position marked ? to see if repetitions occur. I claim that there are only 4 solutions:



Easier! - try the numbers 1 to 6 in a similar triangle. How many solutions?

Find the Quadruplets

A relatively easy search. The solutions 231 = 3 x 77, 132 = 3 x 44, 564 = 6 x 94 should be found easily. I wonder how many readers will miss 120 = 2 x 60 ?!

Flying the Flag

Nicely open-ended activity! Many opportunities for invention and the sky's the limit - especially if we break away from straight lines.

From *Puzzle Papers in Arithmetic* (adapted)

The cumulative payments after the first few half-year periods are respectively

a) 10000, 20500, 31500, 43000, 55000, 67500, .

b) 10000, 20000, 31000, 42000, 54000, 66000, .

Safe Primes

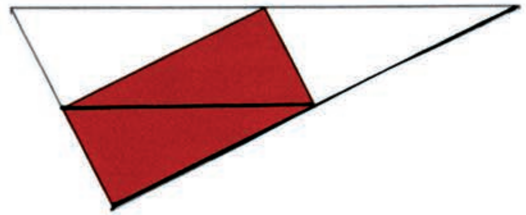
2 x 2 + 1 = 5, 2 x 3 + 1 = 7,
2 x 5 + 1 = 11 2 x 11 + 1 = 23,
2 x 23 + 1 = 47, 2 x 29 + 1 = 59,
2 x 41 + 1 = 83

Not a Word!

I thought it would be amusing to say nothing here but I will just mention that this is my practical paper-and-scissors variation on a proof of Pythagoras' theorem by Leonardo da Vinci.

What Fraction?

Very clearly one half:



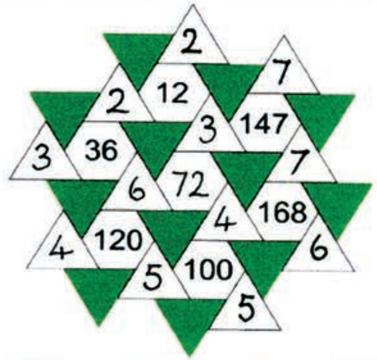
A Cross-Country Challenge

If one cycles for x hours the other will cycle for $2x$ hours: then $12(x + 2x) = 24$ and x is two-thirds of an hour, i.e. 40 minutes. The faster walker cycles 8 km in 40 mins and walks 16 km in 2h 40m. The slower walker walks 8 km in 2 hours and cycles 16 km in 1h 20m. The total time is 3h 20m. This works whichever cycles first.

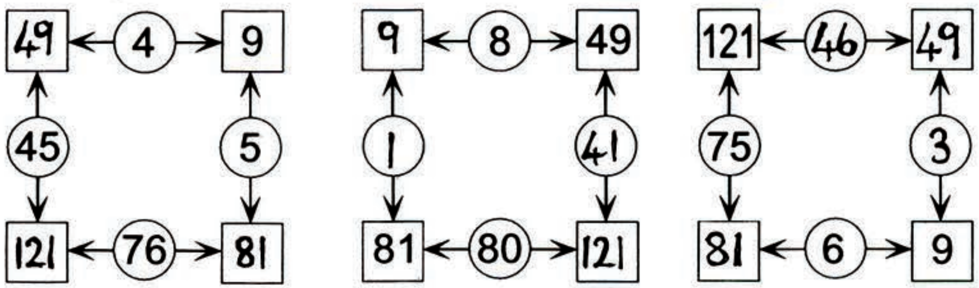
Small Puzzle

2	3
5	6

Factors



Perfect Squares



Discounting rotations and reflections - and taking for granted that our numbers are eight different natural numbers - there are altogether eight 'Perfect Squares' with the corners 9, 49, 121 and 81 : the number between the 9 and the 49 can be 1, 2, 3, 4, 5, 6, 7, or 8.